

# Trade Disclosure and Strategic Behavior in a Three Stage Inter-Dealer Model\*

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## Abstract

The paper examines an inter-dealer model where the disclosure of information is modeled as noisy informative signals. We consider an additional inter-dealer channel of negotiation and let an informed market-maker optimize offsetting his position through both rounds of trade. The results challenge market regulators policy recommendations on more disclosure of information and the risk sharing measured by the volume traded in the first round decreases with the precision of information in both inter-dealer stages. When we endogenize the precision of the third round informative signal allowing for a threshold volume in the second round that triggers a more precise signal in the third round, the risk sharing will be even lower as both the public investor and the winning dealer play strategically to avoid revealing further information.

## JEL Codes:

**Keywords:** Inter-Dealer Trading, Trading Disclosure, Strategic Trading, Linear Equilibria, Risk-Sharing.

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# 1 Introduction

Most of the dealership models in the market microstructure literature assume a single dealer market rather than competitive multiple dealers. The literature concentrates on equity markets, and particular the NYSE, a specialist (single dealer) market. However, other important equity markets such as NASDAQ and the London Stock Exchange (LSE), as well as government securities markets in most developed countries, are structured as multiple dealer markets<sup>1</sup>. In this paper we explore a multiple dealership environment where trades take place in three consecutive stages. Dealers will compete for the initial trade and the winning dealer will use the inter-dealer market to offset his inventory in two consecutive opportunities. We let noisy informative signals be revealed in each stage and evaluate the risk-sharing and welfare implications conditioned on the precision of the signals. The asymmetry of information will be key reason here behind trades. Precision parameters can also be made endogenous and informed dealers will play strategically to avoid revealing information.

As opposed to a single dealer setup, multiple dealers will compete for their share on the customer's order flow, for example a Hedge Funds, which can be informative concerning the asset's final payoff. Trades between customers and dealers will take place in a public trading environment. The existence of an inter-dealer market offers for each dealer the option of managing their inventory via transactions with each other. Indeed, in the inter-dealer market dealers have the choice of either trading bilaterally with each other or indirectly via an inter-dealer broker system. Ho and Stoll (1983), Leach and Madhavan (1993), Vogler (1997) and Naik, Neuberger and Viswanathan (1999), among others, work with multi-dealer models. None of them, however, has proposed a multi-stage inter-dealer setup. Ho and Stoll (1983) was the first to model inter-dealer trading, but allowed customer trades and inter-dealer trades to be carried out in an identical manner. As most of the articles that followed, they assume a high degree of transparency in the trading environment and that transaction and dealer inventories are public information<sup>2</sup>. Viswanathan and Wang (2004) is the only paper we found that works with several inter-dealer stages of trade. They extend the traditional single inter-dealer round of trade to a sequential auction with multiple rounds of unit-auctions. Results show that liquidity falls and the seller is better off with more rounds of auctions, rationalizing the use of sequential auctions and explaining the phenomenon of "hot potato" trading.

This paper intends to revisit the topic and discuss an alternative scheme for the information disclosure process in a more realistic multi-stage inter-dealer market structure. We propose a three stage model where the market making sector possess two inter-dealer rounds to manage his position after a initial public trade. Our model is based on the two stage model proposed by Naik, Neuberger

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<sup>1</sup>Gravelle (2002) examines the structural differences between multiple dealer equity markets and government securities.

<sup>2</sup>Discussing the empirical results of foreign exchange market models Lyons (1996a) states tha: "a microstructural understanding of this market requires a much richer multiple-dealer theory than now exists"

and Viswanathan (1999). The extension we propose is made to accommodate, in a certain way, real direct trading and a broker's screens channels of negotiation. Instead of the sequential auction model proposed by Viswanathan and Wang (2004), we prefer to include only one additional stage to work as the broker's screens channel just mentioned and focus on the information revealing process and the strategic behavior on those trades. The asymmetry of information will be the key reason here behind trades. The motivation differs from the usual inventory models in the literature where dealers charge fees in the form of bid-ask spreads to provide liquidity. Ho and Stoll (1983) and .

Another important assumption of the model is that the information revealing process is noisy and individuals are not able to learn perfectly from the moves of other individuals. Although not standard in the literature, we believe this framework is a very realistic description of the trading environment. We follow Angeletos and Werning (2006) and admit that it captures in a parsimonious way the idea that players revise their beliefs based on others' actions and that it may take more than one period to invert the full information content<sup>3</sup>. Indeed, the two inter-dealer stages will be strategically used by informed players to avoid revealing information and extract rents from uninformed parties.

We depart from the base inter-dealer literature and let a public investor receives an informative signal that induces the trade with one out of the many dealers operating in a given market making sector. Dealers will compete for the initial trade as this reveals public investor's private information concerning the asset's payoff that can be exploited in subsequent rounds of trade. The winning dealer will offer liquidity for private investors using proprietary inventory and use the inter-dealer market to offset his positions. Dealers who do not win the initial trade are assumed to play competitively in both inter-dealer rounds and are only assumed to trade with the winning dealer. We do also allow information in the model to be released as noisy informative signal as in Dasgupta (2006) where the precision parameter can be controlled and used to run alternative scenarios. When compared with the usual extreme full disclosure or non-informative assumptions as in Naik, Neuberger and Viswanathan (1999), this modification makes it possible to simulate how trades evolve between different channels of transaction and the way risk-sharing in the economy changes as the asymmetry of information varies. As far as we know, no other multi dealership model has successfully managed to capture asymmetry of information and strategic behavior in a two-stage inter-dealer market.

The risk sharing here will depend on the asymmetry of information as dealers who did not win the initial trade will be less willing to hold the risky asset the noisier the signal they receive in both rounds of trades. The volume traded will be higher the greater the differential of information among players. Simulations are conducted under an exogenous and an endogenous setup. When we allow a certain threshold volume traded in the first inter-dealer stage to trigger a higher precision parameter in the final round, both public investor and winning dealer will play strategically to restrict trades in the second round. The idea here is

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<sup>3</sup>Minelli and Polemacharkis (2003) work similar theme.

that dealers infer from the amount of transaction but it takes one period to process information and actually use this in the final round. In general, this endogenous setup limits the winning dealer capacity to extract rents from the remaining dealers and thus reduces his willingness for trade in the initial round. On the other hand, the public investor anticipates the transactions that take place in inter-dealer stages and the risk shearing measured by the volume traded in the first period further decreases.

The results we obtain contribute to the discussion on policy recommendations on transparency and trade disclosure on both equity and government securities dealer markets.

This paper is organized as follows. In section 2, we develop an exogenous precision model and solve for the equilibrium in each one of the three stages considered. Section 3 simulate a numerical example for the base model and discuss the effects of different precision parameters and implied asymmetry of information in equilibrium. In section 4 we propose an endogenous precision model and show how the problem changes when players can play strategically to avoid the revelation of information. Section 5 concludes.

## 2 Base Model

The base model is built on the Two-Stage Model proposed by Naik, Neuberger and Viswanathan (1999). We let a Public Investor (*PI*) receives a noisy informative signal concerning the payoff of a given asset and quote a trade with one of the  $K + 1$  identical dealers (or market makers) that operate in an inter-dealer market. Both *PI* and the dealers are risk averse and for simplification reasons have zero starting inventory, such that short-sales will be allowed without restriction. Dealers will compete for the initial trade to gather information that lead to an informational advantage that can be exploited in future rounds of trade. We let  $MM_0$  denotes the winning dealer.

The inter-dealer market is open for trade in two consecutive opportunities. Right after the initial trade  $MM_0$  can trade directly with other market participants, and some information possessed by  $MM_0$  is revealed as a noisy signal. A third and final round is open for trade in broker's screens. This will be  $MM_0$ 's last opportunity to offset his position in the inter-dealer market before the asset's terminal payoff. The same  $K$  remaining dealer will be willing to trade once again as the information conditioning their beliefs will be made upon an additional informative noisy signal generated in the screens. The trade-off between both channels of trade comes from the informational content and the optimization problem and pricing rule applied by the remaining dealers while trading with  $MM_0$ . This point will be made clear as we now go further in the model.

The information process that determines the asymmetry of information is built on a key assumption concerning the precision of the informative signals generated in each stage. We make this precision purely exogenous in the base formulation of the model and evaluate the effects of greater transparency in equilibrium. In section 4 we show an extension to the baseline where the pre-

cision of the informative signal in the final round will depend on the volume traded in the previous round.

*A. Setup*

*Stage 1: Public Investor receives informative signal and goes for a trade*

*PI* receives an informative signal  $v = w + \frac{1}{\varkappa}\epsilon$  regarding the payoff  $w$  of the asset, where  $\epsilon \sim N(0, 1)$  and  $\varkappa$  is the precision parameter. The public investor is assumed to have a CARA utility function with absolute risk aversion coefficient of  $\theta$  and have zero inventory before to the initial trade. A positive signal motivates the initial trade as the public investor wishes to buy a fraction  $y$  of shares that balances the expected return and the risk associated with the updated conditional distribution on the payoff.

*PI* is able to quote for a trade with  $K + 1$  identical dealers that operate in the inter-dealer market but is allowed, for simplicity, to trade with a single dealer only. We let this initial trade to be fully informative for the winning dealer regarding the signal received by *PI*.

*Stage 2:  $MM_0$  trades  $q_2^D$  and the remaining dealers set competitive  $p_2$ .*

The first opportunity for  $MM_0$  to offset his position after the initial trade will be through direct trading with the remaining  $K$  dealers in the market. We call this the second round of trade. Some information will be revealed here for the remaining dealers within the strategy adopted by the winning dealer and the quantity traded. We assume that the market can infer relevant information  $v$  received by  $MM_0$  in the first stage as a noisy signal  $s_i(v, \tau)$ , where the precision  $\tau$  is made a variable of the model<sup>4</sup>. The remaining dealers will update their conditional distribution of the payoff of the share based on this signal and set each one a linear price schedule  $p_2$  for the fraction of trade received<sup>5</sup>. Since all the remaining dealers are identical we let each receive an equal share of the trade in the second round. They will behave competitively, optimize under a CARA utility function with absolute risk aversion coefficient of  $\gamma$  and have zero inventory and receive their reservation utility while trading in the second round.

*Stage 3:  $MM_0$  trades  $q_3^S$  and the remaining dealers set competitive  $p_3$ .*

The winning dealer does not need to offset his entire position in the second round. We consider a third and final round that will take place in broker's screens. The existence of an additional stage of trading allows him to balance his adjustments between both inter-dealer stages. Once again, we assume that the remaining dealers will behave competitively in the final round and do not consider the trade made in the previous round as inventory. The remaining are also only allowed to trade with  $MM_0$ . The basic structure here is the same and the major difference will be an additional informative signal  $s_i(v, \delta)$  that we let update the second stage conditional distribution and which will cause an inevitably lower asymmetry of information in the third round. Even though

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<sup>4</sup>Instead of considering extreme assumptions for the learning process like perfect learning or no learning at all, the signal structure we choose is able to capture the dynamics of the model for a wide range of informational asymmetry. (The advantages of this setup will be made clear later on in the paper)

<sup>5</sup>We show later that a linear equilibrium exists and assure a linear dependence of the informative component  $z$  that will be updating the remaining dealers beliefs.

would prefer deals in a less informative second round,  $MM_0$  faces a trade-off between the two channels due to the increasing linear pricing schedule considered by the remaining dealers.

Table 1 shows the main variables of the model.

**Table1:** Variables of the Model

<i>Variable</i>	<i>Description</i>
$w$	True value of asset payoff
$y$	Buying order placed by IP in the first round of trade
$\gamma$	$MM_0$ 's coefficient of risk aversion
$\theta$	$PI$ 's coefficient of risk aversion
$p_1$	Price in stage 1 (public trade)
$p_2$	Price in stage 2 (direct trading)
$p_3$	Price in stage 3 (broker's screens)
$q_2^D$	Quantity traded by $MM_0$ in the second stage
$q_3^S$	Quantity traded by $MM_0$ in the third stage
$K + 1$	#dealers in inter-dealer market
$\sigma_I^2$	$PI$ and $MM_0$ updated estimate of variance of the payoff
$\sigma_w^2$	Asset's variance of the payoff
$\chi$	$PI$ 's informative signal precision parameter
$\tau$	Direct Trading precision parameter
$\delta$	Broker's Screens precision parameter

### *B. Solving the Game*

We solve the model by backward induction. Starting from the last round of trade,  $MM_0$  takes the prices and quantities traded in the previous rounds as given and solve for the final volume he wishes to buy back. In the second round  $MM_0$  takes once again as given the equilibrium in the first round round, but incorporates the equilibrium in the final round and consider  $q_3^S$  as a reaction functions of the choices in the second round. In both inter-dealer stages the remaining dealers react setting competitive prices.

In the first stage  $MM_0$  will now anticipate the game that follows in the inter-dealer market and set a competitive price  $p_1$  for the share  $y$  brought by the public investor in terms of the equilibrium prices and quantities in the second and third stages. The public investor will solve an equivalent problem and set the amount  $y$  that he wishes to buy. His optimization problem will assume the price function defined by  $MM_0$  that ultimately depends on the volume  $y$  itself, and so the strategic move defined by  $PI$  in this setup will be essential for the entire dynamics of the model.

## **2.1 First Stage Winning Dealer Optimization Problem**

The winning dealer ( $MM_0$ ) optimization problem is choosing the volumes he wishes to buy back through both direct ( $q_2^D$ ) and Inter-Dealer broker screens ( $q_3^S$ ) channels after selling  $y$  quotes in the first stage of trading for the public

investor ( $PI$ ). The initial trade generates a positive cash flow for  $MM_0$  in the first round while the volumes bought back in the inter-dealer market represent costs and the terminal payoff will depend on the effective shares he ends up carrying after the last opportunity to trade in the third stage. Conditioned on the full informative signal revealed by the public trade to  $MM_0$  the problem can be stated as:

$$E(U_0) = -E \left[ \exp \left( - \left( (q_3^S + q_2^D - y) w + yp_1 - q_3^S p_3 - q_2^D p_2 \right) \mid v \right) \right]$$

Alternatively, the problem can be equivalently written under a mean-variance representation:

$$\begin{aligned} & -\frac{1}{\gamma} \log (E [U_0(q_3^S, q_2^D) \mid v]) \\ &= E \left[ (q_3^S + q_2^D - y) w + yp_1 - q_3^S p_3 - q_2^D p_2 \mid v \right] \\ & -\frac{\gamma}{2} \text{Var} \left[ (q_3^S + q_2^D - y) w + yp_1 - q_3^S p_3 - q_2^D p_2 \mid v \right] \\ &= (q_3^S + q_2^D - y) v + yp_1 - q_3^S p_3 - q_2^D p_2 \\ & \quad -\frac{\gamma}{2} (q_3^S + q_2^D - y)^2 \sigma_I^2 \end{aligned}$$

where  $E(w/v) = v$  and  $\text{Var}(w/v) \equiv \sigma_I^2$  represent the conditional distribution of the true values of the shares given the perfect information assumption provided by the public trade. We follow addressing the problem by backward induction.

### 2.1.1 Inter-Dealer Broker Screens (Third Stage)

$MM_0$  problem on the third stage will be maximize his utility with respect to  $q_3^S$ . Taking as given the variables defined in the previous rounds of trade and once again conditioning in the information revealed by the public trade:

$$\begin{aligned} & -\frac{1}{\gamma} \log (E [U_0(q_3^S) \mid v]) \tag{1} \\ &= (q_3^S + q_2^D - y) v + yp_1 - q_3^S \left( \alpha_3 + \beta_3 \frac{q_3^S}{K} \right) - q_2^D p_2 \\ & \quad -\frac{\gamma}{2} (q_3^S + q_2^D - y)^2 \sigma_I^2 \end{aligned}$$

We assume that the  $K$  remaining dealers apply a linear price rule  $p_3 = \alpha_3 + \beta_3 q_{3,i}^S$ , where  $q_{3,i}^S = \frac{q_3^S}{K}$ . It will be shown later that a simple price schedule like that will generate an equilibrium. The first order condition here with respect to  $q_3^S$  is:

$$\begin{aligned} q_3^S &= \frac{K}{2\beta_3 + K\gamma\sigma_I^2} (v - \alpha_3 - \gamma\sigma_I^2 (q_2^D - y)) \tag{2} \\ q_{3,i}^S &= \frac{1}{2\beta_3 + K\gamma\sigma_I^2} (v - \alpha_3 - \gamma\sigma_I^2 (q_2^D - y)) \end{aligned}$$

or equivalently:

$$q_3^S = K\Psi (v - \alpha_3 - \gamma\sigma_I^2 (q_2^D - y)) \quad (3)$$

$$\Psi = \frac{1}{2\beta_3 + K\gamma\sigma_I^2}$$

### 2.1.2 Direct Trading (Second Stage)

In the second stage  $MM_0$  optimization problem will take into account the equilibrium in the final round and take as given the initial trade such that:

$$-\frac{1}{\gamma} \log (E [U_0(q_2^D, q_3^S(q_2^D)) | v]) \quad (4)$$

$$= (q_3^S + q_2^D - y) v + yp_1 - q_2^D \left( \alpha_2 + \beta_2 \frac{q_2^D}{K} \right)$$

$$- q_3^S \left( \alpha_3 + \beta_3 \frac{q_3^S}{K} \right) - \frac{\gamma}{2} (q_3^S + q_2^D - y)^2 \sigma_I^2$$

Once again, considering a linear price schedule for the trades conducted outside screens  $p_2 = \alpha_2 + \beta_2 q_{i,2}^D$ . The first order condition or  $q_2^D = K q_{i,2}^D$ , substituting for  $q_3^S$  obtained in the following stage:

$$v [1 + \Omega] - \alpha_2 - \frac{2\beta_2 q_2^D}{K} - \Omega \alpha_3 - \frac{2\beta_3 \Omega q_3^S}{K} \quad (5)$$

$$- \gamma \sigma_I^2 (q_3^S + q_2^D - y) (1 + \Omega) = 0$$

where;

$$\frac{dq_3^S}{dq_2^D} = -K\Psi\gamma\sigma_I^2 = \Omega \quad (6)$$

The equation that defines  $q_2^D$  as a function of the exogenous variables of the model can be written as:

$$q_2^D = \Phi (\alpha_2 + \Omega \alpha_3 - v (1 + \Omega) - 2\gamma\sigma_I^2 \Psi \beta_3 q_3^S + \gamma\sigma_I^2 (1 + \Omega) (q_3^S - y)) \quad (7)$$

$$\Phi = \frac{-K}{2\beta_2 + K\gamma\sigma_I^2 (1 + \Omega)}$$

### 2.1.3 Public Investor Initial Trade (First Stage)

In the first round of trade we assume that all the negotiating power resides with the public investor. The  $K + 1$  dealers in the market will compete for the initial trade such that all the rents provided by the asymmetry of information will be

captured by  $PI$ . The expected utility of  $MM_0$  equals:

$$\begin{aligned}
& -\frac{1}{\gamma} \log (E [U_0(p_1(q_2^D, q_3^S, y) | v)]) \\
& = (q_3^S + q_2^D - y) v + y p_1 - q_3^S \left( \alpha_3 + \beta_3 \frac{q_3^S}{K} \right) - q_2^D \left( \alpha_2 + \beta_2 \frac{q_2^D}{K} \right) \\
& \quad - \frac{\gamma}{2} (q_3^S + q_2^D - y)^2 \sigma_I^2 = 0
\end{aligned} \tag{8}$$

which yields the first stage price that reflects the ability of  $MM_0$  to manage his position and exploit the informational advantage in the inter-dealer market:

$$p_1 = \frac{1}{y} \left[ q_3^S p_3 + q_2^D p_2 + \frac{\gamma}{2} (q_3^S + q_2^D - y)^2 \sigma_I^2 - (q_3^S + q_2^D - y) v \right] \tag{9}$$

## 2.2 The $K$ Remaining Dealers Problem

We address the optimization problem of the  $K$  others market makers assuming a competitive behavior in both rounds of inter-dealer trading<sup>6</sup>. They will condition their information on different informative signals about the asset's payoff, inverted with some noise from the orders received in each inter-dealer stage. In the base model we treat the precision of these signals purely exogenous and let an alternative endogenous model for Section 3. Conditioned on the information received and the updates made, the inter-dealer market will solve for a similar maximization problem and determine the price schedule in both rounds.

### 2.2.1 Updating Beliefs

It follows from (3) and (7) that the volumes traded in the second and third round depend linearly on the informational content  $v = w + \frac{1}{\lambda} \epsilon$  received by  $MM_0$  in the first round. This information will be transmitted through the transactions conducted in both following stages with some noise. We let the technology that invert this informational content from the quantities be represented by a noisy signal  $s_i$ . In the second stage the precision will be measured by an exogenous parameter  $\tau$  and can represent as:

$$s_i(v, \tau) = v + \frac{1}{\tau} \eta_i \tag{10}$$

where  $\eta_i$  is distributed Standard Normal and independent of  $v$ . The noisy component will add to the signal variance. The inter-dealer market will update his conditional distribution of the payoff  $w$  following a Gaussian signal:

$$\begin{aligned}
E(w | s_i(v, \tau)) &= \mu_w + \varphi_2 (s_i(v, \tau) - \mu_v) \\
Var(w | s_i(v, \tau)) &= (1 - \rho_2^2) \sigma_w^2
\end{aligned} \tag{11}$$

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<sup>6</sup>We follow Subrahmanyam (1991) and assume that each of the remaining dealers receive an equal share of the trade in both rounds of trade and do not extract any rent from the information received in the second round.

where  $\mu_w$ ,  $\mu_v$  and  $\sigma_w^2$  are the initial priors before the trade and:

$$\begin{aligned}\varphi_2 &= \frac{\text{cov}(w, s_i(v, \tau))}{\text{var}(s_i(v, \tau))} = \frac{\sigma_w^2}{\sigma_w^2 + \frac{1}{\kappa^2} + \frac{1}{\tau^2}} \\ \rho_2 &= \frac{\text{cov}(w, s_i(v, \tau))}{\sqrt{\sigma_w^2 \sigma_{s_i}^2}} = \frac{\sigma_w^2}{\sqrt{\sigma_w^2 [\sigma_w^2 + \frac{1}{\kappa^2} + \frac{1}{\tau^2}]}}\end{aligned}$$

In the broker screens we let  $\delta$  represent the precision of similar informative signal:

$$s_i(v, \delta) = v + \frac{1}{\delta} \omega_i \quad (12)$$

where  $\omega$  is also distributed Standard Normal and independent of  $v$ . The update in this stage will be made over the conditional distribution obtained after the second round. Once again, following a Gaussian signal  $s_i(v, \delta)$  the distribution will be:

$$\begin{aligned}E(w | s_i(v, \delta), s_i(v, \tau)) &= E(w | s_i(v, \tau)) + \varphi_3 (s_i(v, \delta) - E(v | s_i(v, \tau))) \quad (13) \\ \text{Var}(w | s_i(v, \delta), s_i(v, \tau)) &= (1 - \rho_3^2) \sigma_{w/s_i(v, \tau)}^2\end{aligned}$$

such that:

$$\begin{aligned}\varphi_3 &= \frac{\text{cov}(w, s_i(v, \delta))}{\text{var}(s_i(v, \delta))} = \frac{\sigma_w^2}{\sigma_w^2 + \frac{1}{\kappa^2} + \frac{1}{\delta^2}} \\ \rho_3 &= \frac{\text{cov}(w, s_i(v, \delta))}{\sqrt{\sigma_w^2 \sigma_{s_i}^2}} = \frac{\sigma_w^2}{\sqrt{\sigma_w^2 [\sigma_w^2 + \frac{1}{\kappa^2} + \frac{1}{\delta^2}]}}\end{aligned}$$

### 2.2.2 Inter-Dealer Trading Prices

The inter-dealer market will set up prices in each stage based on the volume demanded by  $MM_0$  and the updated conditional distributions. In the third stage the remaining dealers apply a linear pricing rule as function of the fraction  $q_{3,i}^S$  brought to the broker screens and negotiated with each one that can be represented as:

$$p_3 = \alpha_3 + \beta_3 q_{3,i}^S \quad (14)$$

Considering  $K$  identical risk averse market makers with negative exponential utility and risk aversion coefficient of  $\gamma$  we solve the problem for a representative dealer with zero starting inventory. We follow Subrahmanyam (1991) and let the inter-dealer market compete for the transaction with  $MM_0$ . The competition contributes for a zero expected utility in the third stage, where both informative signals will be updating the remaining dealer's beliefs. In mean-variance representation and ignoring the outcome of the previous round:

$$-\frac{1}{\gamma} \log E [U_i(p_3(q_3^S)) | s_i(v, \delta), s_i(v, \tau)] = 0 \quad (15)$$

$$q_3^S (p_3 - E(w | s_i(v, \delta), s_i(v, \tau))) - \frac{1}{2} \gamma (q_3^S)^2 \text{Var}(w | s_i(v, \delta), s_i(v, \tau)) = 0$$

The price in the third stage can be implicitly defined as:

$$p_3 = \frac{1}{2}\gamma q_3^T Var(w | s_i(v, \delta), s_i(v, \tau)) + E(w | s_i(v, \delta), s_i(v, \tau)) \quad (16)$$

Comparing (16) and (14) we may write  $\alpha_3$  and  $\beta_3$  in terms of the conditional distribution:

$$\begin{aligned} \alpha_3 &= E(w | s_i(v, \delta), s_i(v, \tau)) \\ \beta_3 &= \frac{K}{2}\gamma Var(w | s_i(v, \delta), s_i(v, \tau)) \end{aligned} \quad (17)$$

In the direct trading round  $MM_i$  will apply an equivalent pricing rule:

$$p_2 = \alpha_2 + \beta_2 q_{i,2}^D \quad (18)$$

The expected utility here will be conditioned in a single informative signal where the updates are given by (11). Once again, the competition contributes for a zero expected utility in mean-variance representation:

$$q_2^D (p_2 - E(w | s_i(v, \tau))) - \frac{1}{2}\gamma (q_2^D)^2 Var(w | s_i(v, \tau)) = 0 \quad (19)$$

such that the price set by the each of the remaining dealers for their fraction of the trade is:

$$p_2 = \frac{1}{2}\gamma q_2^D Var(w | s_i(v, \tau)) + E(w | s_i(v, \tau)) \quad (20)$$

and the components  $\alpha_2$  and  $\beta_2$  as a function of the conditional distribution in the second round:

$$\begin{aligned} \alpha_2 &= E(w | s_i(v, \tau)) \\ \beta_2 &= \frac{K}{2}\gamma Var(w | s_i(v, \tau)) \end{aligned} \quad (21)$$

### 2.3 The Public Investor Problem

We endogenize the public trade size  $y$  modeling the public investor's trading strategy. We endow the investor with an informative noisy signal  $v = w + \frac{1}{\varkappa}\epsilon$  regarding the payoff of the shares, where  $\epsilon \sim N(0, 1)$  and  $\varkappa$  plays the precision. No initial inventory is considered.  $PI$  will condition his beliefs on the informative signal  $v$ , update  $E(w/v) = v$  and  $Var(w/v) \equiv \sigma_I^2$  and maximize a negative exponential utility with risk aversion coefficient of  $\theta$ .

$$\begin{aligned} Max_y - \frac{1}{\theta} \log E[U(y) | v] \\ = y(v - p_1) - \frac{\theta}{2} y^2 \sigma_I^2 \end{aligned} \quad (22)$$

This informative signal will update  $MM_0$ 's beliefs such that the price  $p_1$  offered for trade in the first round will incorporate  $v$  and must be such that in competition ensures an expected utility for the winning dealer just as high as the expected utility for the market maker who does not get the trade. Any excess would be enough for one of the  $K$  remaining dealers to offer a slightly better price in the first round and win the trade. Considering identical market makers this competition for the trade will lead for a zero expected utility for the winning dealer in the entire three stage game that follows. Taking the price  $p_1$  offered by  $MM_0$  in the first round, conditioned on the volume  $y$  demanded by  $PI$  and defined in (9), the first order condition will be:

$$\begin{aligned}
v - p_1 - \frac{dq_3^S}{dy} \left( \alpha_3 + \frac{2\beta_3}{K} \right) - \frac{dq_2^D}{dy} \left( \alpha_2 + \frac{2\beta_2}{K} \right) & \quad (23) \\
-\gamma\sigma_I^2 (q_3^S + q_2^D - y) \left( \frac{dq_3^S}{dy} + \frac{dq_2^D}{dy} - 1 \right) & \\
+ \left( \frac{dq_3^S}{dy} + \frac{dq_2^D}{dy} - 1 \right) v - \theta\sigma_I^2 = 0 &
\end{aligned}$$

where we consider a fully revealing assumption in the public trade such that:

$$\begin{aligned}
\frac{dq_3^S}{dy} &= K\Psi\gamma\sigma_I^2 \left( 1 - \frac{dq_2^D}{dy} \right) \\
\frac{dq_2^D}{dy} &= \Phi \frac{\gamma\sigma_I^2 (1 + \Omega) (K\Psi\gamma\sigma_I^2 - 1) - 2K\Psi^2\gamma^2\beta_3\sigma_I^4}{1 - \Phi K\Psi\gamma^2\sigma_I^4 (2\Psi\beta_3 - 1 - \Omega)}
\end{aligned}$$

### 3 Comparing Equilibria for Different Signal's Precision Scenarios

In order to highlight  $MM_0$ 's optimal choices among different informational asymmetry we run numerical simulations considering sufficiently wide ranges for the precision parameters to describe the dynamics of the base model. The simulations help us to understand how all these variables interact and the equilibrium evolves under different assumptions for the two exogenous precision parameters in inter-dealer market. We calibrate the model for a set values commonly find in equivalent calibrations in the literature:  $\gamma=0.20$ ,  $\theta=0.20$ ,  $K=20$ ,  $v=10$ ,  $\sigma_w^2=1.00$ ,  $\mu_w=\mu_v=5$ ,  $\chi=10$ ,  $\tau=1$  and  $\delta=2$ . Results do not change significantly when we chose different set of calibrations.

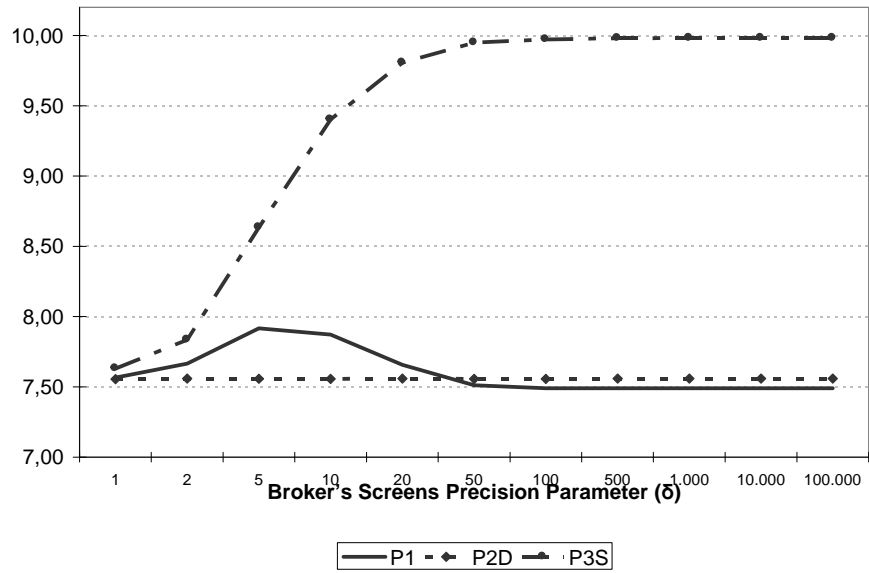
#### 3.1 Inter-Dealer Broker's Screens Precision Parameter ( $\delta$ )

Figure 1 examines the relationship between transaction prices in all stages of trading. The contribution of the precision parameter  $\delta$  will be straightforward in the third stage as the informative signal is directly related to the price formation in the final round through the updates made by the remaining dealers. On the

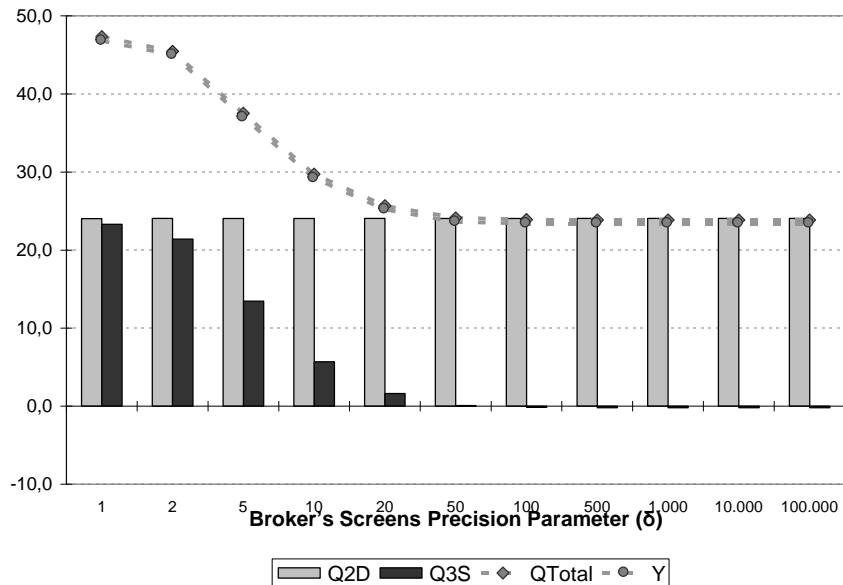
other side, the effects on the decisions made in the first and second round come from the strategic behavior of all the players considered. Solving the model by backward induction we allow players in the preceding rounds to incorporate the conditions met in the final round to their plans of trading.

Third's round price curve will be increasing in the precision parameter, becoming flat for sufficiently large  $\delta$ . The updates made over more precise informative signals reduce the risk premia perceived by the remaining dealers and contribute for an upward pressure in prices. The volume left for trade in the final round decreases with the precision, becoming slightly negative ( $MM_0$  selling) as the asymmetry of information vanishes and the risk taking profile considered for the winning dealer ( $\gamma=0.20$ ) dominates. Figure 2 makes the point. The conditional mean in the final round increase in a way that it will not be optimal for  $MM_0$  to split his trades in both channels of trade. The asymmetry in the second round will be such that it will be better to carry almost all the trade in the second round and avoid higher prices set by the remaining dealers in the third round.

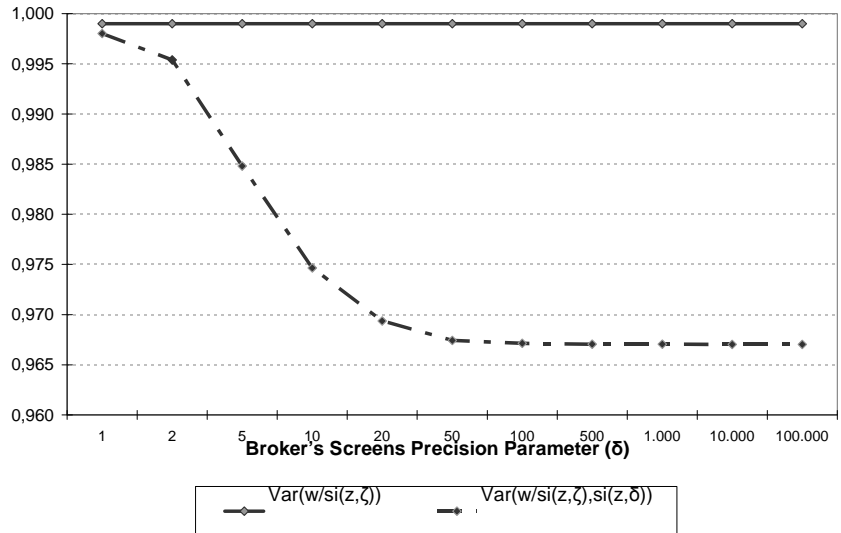
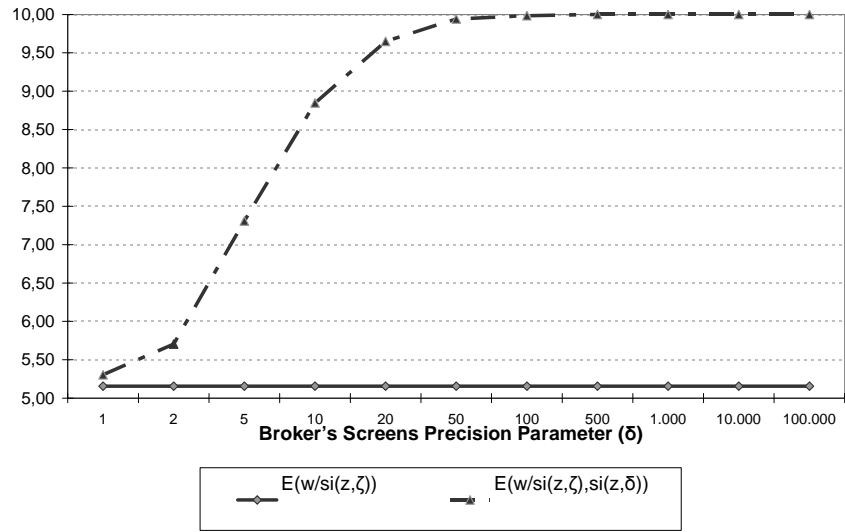
**Figure1:** Transaction Prices versus Broker's Screens Precision Parameter ( $\delta$ )



**Figure2:** Transaction Volumes versus Broker's Screens Precision Parameter ( $\delta$ )



**Figure3:** Payoff's Conditional Distribution versus Broker's Screens Precision Parameter ( $\delta$ )

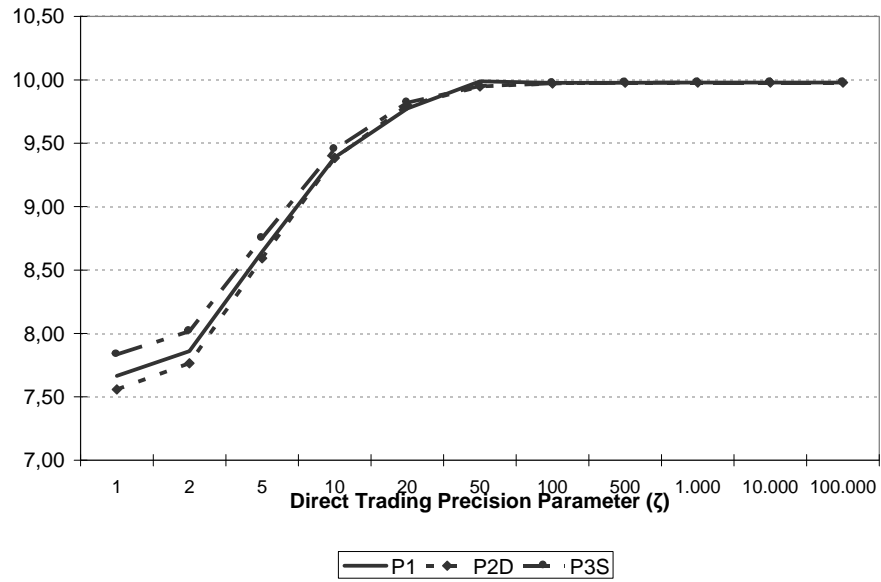


### 3.2 Direct Trading Precision Parameter ( $\tau$ )

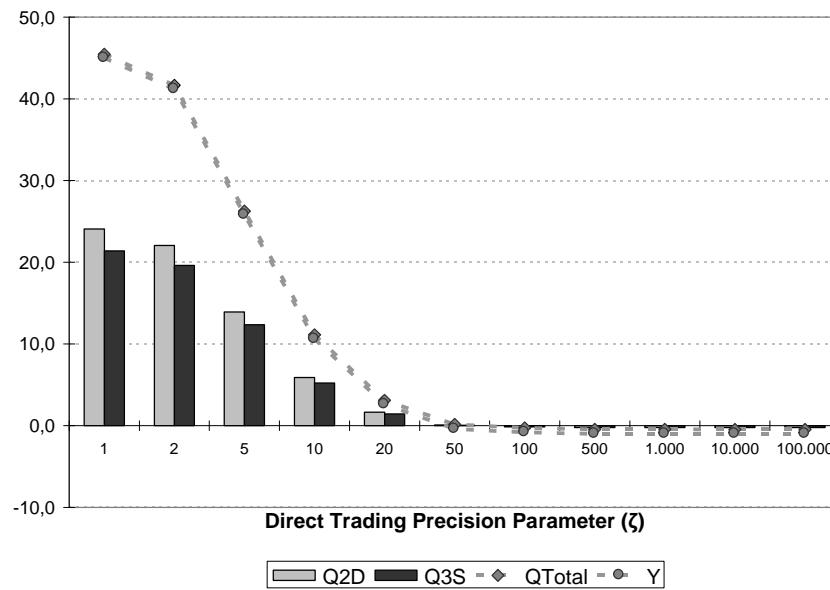
We now simulate the effects of different precision parameters for the signal generated in the second round and that updates the beliefs of the  $K$  remaining dealers in both rounds of inter-dealer trade. The main difference between the precision in the second and third round is that the higher the precision here, the similar the asymmetry in both rounds as the signal in the second round is carried to final round. On the other hand, the greater the precision in the third round, and the lower the precision in the second, the greater the difference in the informational content in the two rounds of trade.

Figure 4 shows the evolution of the prices in all three stages. The asymmetry of information in both inter-dealer's rounds decreases with the precision parameter and the prices converge to the expected payoff value  $E(w/x) = 10$ . Quantities traded in both stages also decrease together as the informational advantage collapses. For sufficiently high precision parameters the asymmetry is almost completely eroded and the trades simply cease to exist. No transaction may take place in the first place.

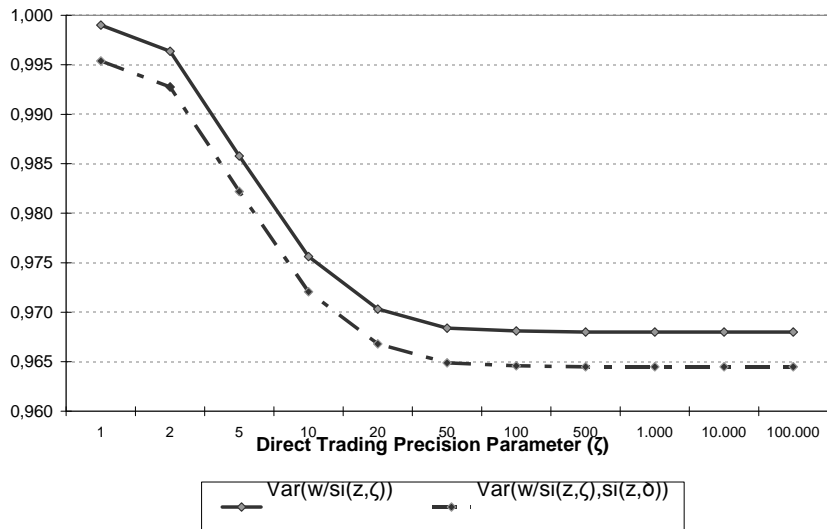
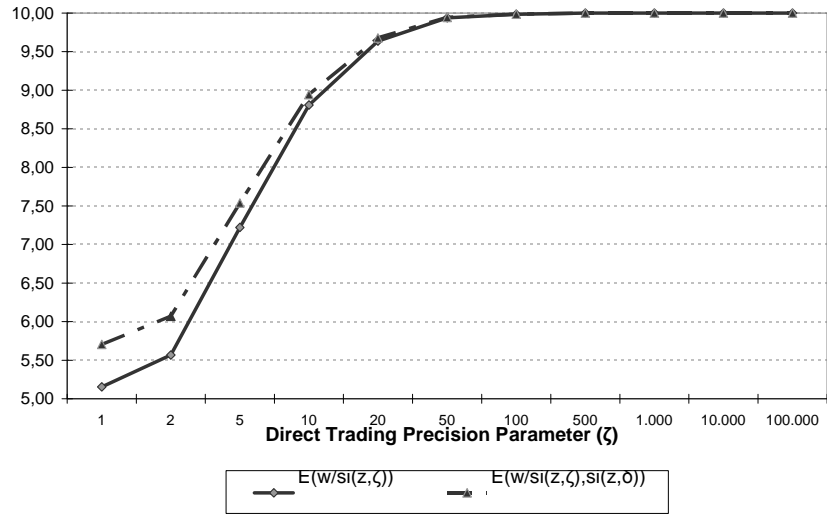
**Figure4:** Transaction Prices versus Direct Trading Precision Parameter ( $\tau$ )



**Figure5:** Transaction Volumes versus Direct Trading Precision Parameter ( $\tau$ )



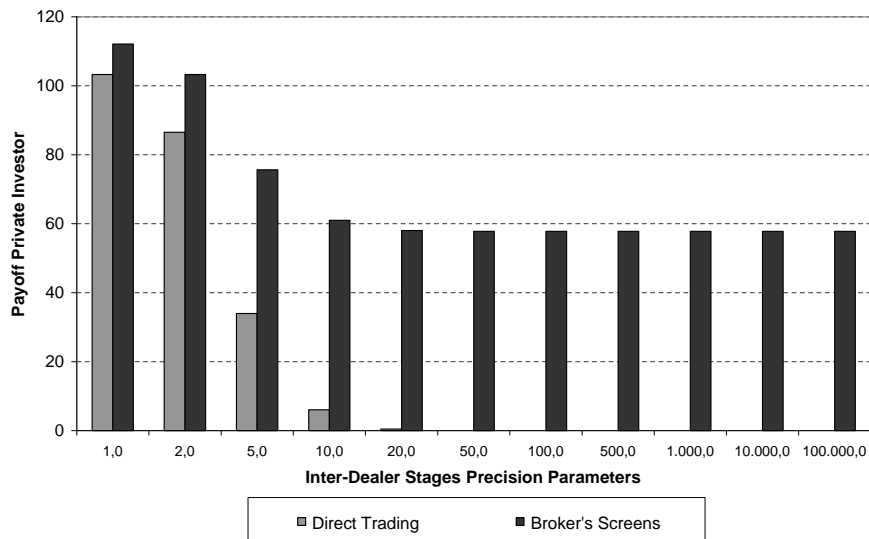
**Figure6:** Payoff's Conditional Distribution versus Direct Trading Precision Parameter ( $\tau$ )



### 3.3 Welfare Comparison

To investigate the welfare implications of different precision parameters scenarios we compare the payoffs derived by the private investor. Assuming that the  $K + 1$  dealers will compete for the trade in the first round of negotiation and derive zero expected utility, all the bargaining power will be with the public investor. The investor ability to extract rents from the winning dealer will be higher the greater the asymmetry of information between the winning dealer and the remaining players in both inter-dealer rounds of trade. The greater the uncertainty about the payoff of the asset, the lower the price uninformed dealers would be willing to sell their inventories. In the first round, the winning dealer will be such that his informational advantage the will be exploit trading with the remaining dealers will be reverted to the private investor.

Figure7: Private Investor Welfare Comparison



## 4 An Endogenous Precision Model

The simulations ran so far shows that the rents for negotiation in the first round of trade is higher the greater the asymmetry of information generated by noisier signals in both inter-dealer trading stages. This asymmetry leads  $MM_0$  to extract surplus from the  $K$  remaining dealers witch, due to competition, are ultimately transferred to  $PI$  in the first round of trade. Thus, the higher asymmetry the more favorable conditions for  $PI$  to buy in the first round and consequently the higher the volume traded.

So far we have assumed that the precision concerning the three informative signals in the model were purely exogenous. We now consider a variation of

the model where the precision in the third stage depends on the volume traded in the previous stage. There will be a threshold volume in the second stage  $\overline{q_2^D}$  above which the precision in broker screens increase to  $N\delta$  ( $N > 1$ ). Both *PI* and  $MM_0$  optimization problems will be constrained by the volume  $\overline{q_2^D}$  that triggers a higher precision in the final stage. The motivation behind the endogenous framework is straightforward. We assume that the volume itself is revealing, once it may induce the remaining dealers to re-trade and generate a self-fulfilling sequence of transactions that could be informative.

As before, we solve the model by backward induction. The resolution will be almost identical to the base model, except that now both *PI* and  $MM_0$  will choose between two optimization problems. In the second stage, after selling  $y$  units  $MM_0$  will have to decide between restricting his trade in the second round and avoid a higher precision in the third round or exceeding the threshold  $\overline{q_2^D}$  and generating a more precise signal in the final round. In the base model the choice of volumes traded in each inter-dealer stage would not impact the information revealed.

The *PI* problem in the first would be even more complex as we assume that his initial demand for trade will necessarily be met by  $MM_0$ . *PI* will need to consider  $MM_0$  optimization problem as this will determine the price  $\widehat{p}_1$  for which he will be willing to trade. In the first round *PI* will consider the strategic trade by  $MM_0$  in the inter-dealer market that follows his initial move and maximize choosing a similar problem between a constrained and an unconstrained maximization.

#### 4.0.1 Inter-Dealer Broker Screens (Third Stage)

We once again address the problem by backward induction. Beginning in the final round, we take as given the equilibrium variables in the preceding rounds and thus  $MM_0$ 's optimization problem when we allow the precision to be dependent of the volume traded in the second round will be:

$$\begin{aligned} \text{Max}_{q_3^S} & - \frac{1}{\gamma} \log (E [U_0(q_3^S)/v]) \\ \text{st : } \delta & \iff q_2^D = \widehat{q_2^D} \leq \overline{q_2^D} \\ N\delta & \iff q_2^D = \widetilde{q_2^D} > \overline{q_2^D} \end{aligned}$$

The optimal choices for  $q_3^S$  conditioned on the volume in the previous round will be:

$$\begin{aligned} \widetilde{q_2^D} > \overline{q_2^D} & \implies \widetilde{q_3^S} = K\widetilde{\Psi} \left( v - \widetilde{\alpha}_3 - \gamma\sigma_I^2\widetilde{q_2^D} \right) \\ \widehat{q_2^D} \leq \overline{q_2^D} & \implies \widehat{q_3^S} = K\Psi \left( v - \alpha_3 - \gamma\sigma_I^2\widehat{q_2^D} \right) \end{aligned}$$

where  $\widetilde{\alpha}_3$  and  $\widetilde{\Psi}$  consider  $N\delta$  instead of  $\delta$ . In general, the endogenous precision scheme will increase the volume traded in the broker screens if the restriction

apply and it becomes optimal for  $MM_0$  not to exceed  $\overline{q_2^D}$ . This is the case we will be interested. When it is optimal to exceed  $\overline{q_2^D}$  and generate greater transparency in the third round, the informational advantage lost will lead inevitably to a reduction in  $q_3^S$ .

**Proposition 1** *When it is optimal for  $MM_0$  to restrict  $\widehat{q_2^D} \leq \overline{q_2^D}$  and avoid  $N\delta$ , the amount negotiated in the final round will be at least as large as it would be in the base model, where there were no such conditions on the precision of the informative signal.*

**Proof.** *If for a given public trade  $y$  the trigger  $\overline{q_2^D}$  does not apply and  $\widehat{q_2^D} < \overline{q_2^D}$ , then the quantity left for the third round will be exactly the same as in the base model. If, however, the initial trade  $y$  is such that it now becomes optimal to restrict the transactions in the second stage and preserve the asymmetry to be exploited in the final round, then the fraction of the position did not offset in the second round will add in the next round. ■*

#### 4.0.2 Direct Trading (Second Stage)

The real optimization problem faced by  $MM_0$  will be in the second round. The outcome here will determine the precision in the following round and ultimately the potential asymmetry of information to be exploited. As before,  $MM_0$  will compare the utilities resulting two maximizations. First, we consider the equilibrium when the volume traded here is bounded by  $\overline{q_2^D}$ , the precision in the third stage is kept  $\delta$  and the third stage output is given by  $\widehat{q_3^S}$ :

$$\begin{aligned} \underset{q_2^D}{Max} \quad & -\frac{1}{\gamma} \log (E [U_0(q_2^D, q_3^S(q_2^D))/v]) \\ \text{st : } & \delta \\ & q_3^S = \widehat{q_3^S} \\ & \widehat{q_2^D} \leq \overline{q_2^D} \end{aligned}$$

The optimal response here will be:

$$\widehat{q_2^D} = \begin{cases} \frac{\Phi(\alpha_2 + \Omega\alpha_3 - v(1 + \Omega) - 2\gamma\sigma_I^2\Psi\beta_3q_3^S}{+\gamma\sigma_I^2(1 + \Omega)(q_3^S - y)} & ; \text{ if } \widehat{q_2^D} \leq \overline{q_2^D} \\ \overline{q_2^D} & ; \text{ if } \widehat{q_2^D} = \overline{q_2^D} \end{cases}$$

The alternative optimization problem that  $MM_0$  consider is letting  $q_2^D > \overline{q_2^D}$ ,

increase the precision to  $N\delta$  and take the third stage output  $\widetilde{q}_2^D$ . This leads to:

$$\begin{aligned} \underset{q_2^D}{Max} - \frac{1}{\gamma} \log (E [U_0(q_2^D, q_3^S(q_2^D))/v]) \\ \text{st : } N\delta \\ q_3^S = \widetilde{q}_3^S \\ \widetilde{q}_2^D > \overline{q}_2^D \end{aligned}$$

which yields;

$$\widetilde{q}_2^D = \widetilde{\Phi} \left( \alpha_2 + \widetilde{\Omega}\widetilde{\alpha}_3 - v \left( 1 + \widetilde{\Omega} \right) - 2\gamma\sigma_I^2\widetilde{\Psi}\widetilde{\beta}_3\widetilde{q}_3^S + \gamma\sigma_I^2 \left( 1 + \widetilde{\Omega} \right) \left( \widetilde{q}_3^S - y \right) \right)$$

#### 4.0.3 Public Investor Initial Trade (First Stage)

All the interactions and information disclosure strategies that take place in the inter-dealer market come down to the first round of trade. We assume that *PI* can fully anticipate the strategic behavior behind the inter-dealer market and use this to optimize in the first round. Once again, the optimal volume  $y$  is determined by comparing the utility derived from different optimization problems. *PI* will chose between an unrestricted maximization, where it becomes optimal for  $MM_0$  to make  $\widetilde{q}_2^D$ , and a restricted problem where the amount  $y$  does not trigger the threshold  $\overline{q}_2^D$  in the second stage of trading.

The restricted problem can be represented as:

$$\begin{aligned} \underset{y}{Max} - \frac{1}{\theta} \log E [U (y) / v] \\ \text{st : } \delta \\ p_1 = \widehat{p}_1 \\ q_2^D = \widehat{q}_2^D \end{aligned}$$

where the price  $p_1$  reflects the strategic behavior by  $MM_0$  that follows the public trade. When  $y$  is such that avoids a higher disclosure of information in the third stage, the price asked by  $MM_0$  will be:

$$\widehat{p}_1 = \frac{1}{y} \left[ \widehat{q}_3^S p_3 + \widehat{q}_2^D p_2 + \frac{\gamma}{2} \left( \widehat{q}_3^S + \widehat{q}_2^D - y \right)^2 \sigma_I^2 - \left( \widehat{q}_3^S + \widehat{q}_2^D - y \right) v \right]$$

and the volume  $\widehat{y}$  defined by the optimization follows implicitly from an equivalent first order condition derived in (23).

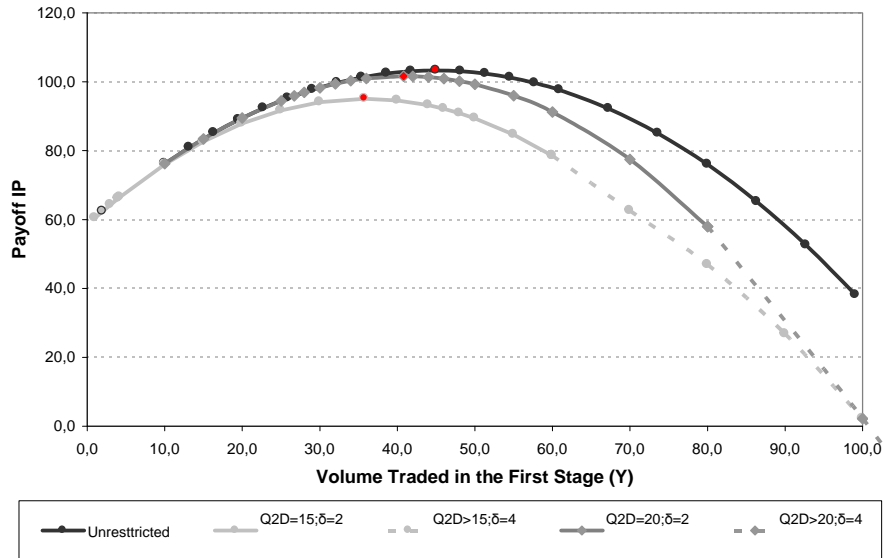
$$\begin{aligned} v - \widehat{p}_1 - \frac{dq_3^S}{dy} \left( \alpha_3 + \frac{2\beta_3}{K} \right) - \frac{dq_2^D}{dy} \left( \alpha_2 + \frac{2\beta_2}{K} \right) \\ - \gamma\sigma_I^2 \left( \widehat{q}_3^S + \widehat{q}_2^D - \widehat{y} \right) \left( \frac{dq_3^S}{dy} + \frac{dq_2^D}{dy} - 1 \right) \\ + \left( \frac{dq_3^S}{dy} + \frac{dq_2^D}{dy} - 1 \right) v - \frac{\theta}{2} \sigma_I^2 = 0 \end{aligned} \quad (24)$$

The unrestricted problem  $\tilde{y}$  will be identical to the base model, except that the precision will now increase to  $N\delta$ . The threshold equilibrium  $\overline{q_2^D}$  will always be triggered whenever  $y$  is high enough to make it optimal to share the trade between both rounds. As mentioned before, we focus on situations where will be optimal to condition the volume traded to restrict the information revealed in the last stage of trade, such that the outcome of the unrestricted problem will not be optimal for the parameters we set. We can always set  $\overline{q_2^D}$  and  $N$  to make it optimal for  $PI$  to restrict the volume traded in the first round.

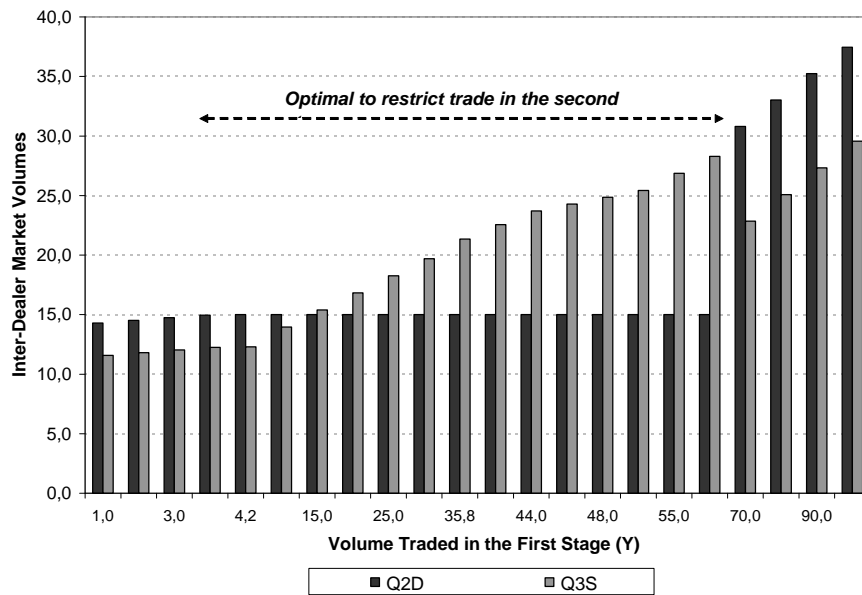
In order to highlight the effects of an endogenous precision model to the risk sharing in the economy we consider a numerical example by specifying the following parameter values:  $\gamma=0.20$ ,  $\theta=0.20$ ,  $K=20$ ,  $v=10$ ,  $\sigma_w^2=1.00$ ,  $\mu_w=5$ ,  $\mu_z=5$ ,  $\chi=10$ ,  $\tau=1$ ,  $\delta=2$  and  $N=2$ . Figure 8 presents the relationship between payoff and volume traded in the first round for  $PI$  unrestricted (base model),  $\overline{q_2^D}=20$  and  $\overline{q_2^D}=15$ . The risk sharing in the economy measured by the initial public trade is lower when we set a binding threshold  $\overline{q_2^D}$  in the second round. More than that, the initial trade decreases the lower the threshold  $\overline{q_2^D}$ . For the range of  $y$  simulated it will always be optimal for  $MM_0$  to avoid a greater transparency in the final round when the threshold is  $\overline{q_2^D}=20$ . Once we set  $\overline{q_2^D}=15$  the restricted optimization will only be optimal up to  $y < 60$ . If the initial trade is made greater than that, it will become optimal for  $MM_0$  to chose  $\tilde{q_2^D}$  even if it generates a more precise signal in the next round.

Figure 9 shows the range of  $y$  that makes it optimal for  $MM_0$  to restrict the volume in the second round to  $\widehat{q_2^D} = \overline{q_2^D}$  and keep the signal less informative.  $MM_0$  will partially compensate in the final round the volume did not trade in the previous round. When  $y$  is high enough to make it worth to exceed  $\overline{q_2^D}$  and reduce the asymmetry, the volume in the second round will then abruptly increase to extract the most from the second round and to compensate for the worst environment in the third round.

**Figure8:** Payoff  $PI$  versus Volume Traded in the First Stage



**Figure9:** Inter-Dealer Market Optimal Volumes  
(Direct and Broker's Screens Trading when  $\overline{q_2^D}=15$ )



## 5 Conclusion

In this paper we propose a multi-stage inter-dealer model where trades in the inter-dealer market are conducted in two sequential stages. Dealers will compete for the public trade in the first round, which will reveal some private information. The winning dealer will be able to offset his unbalanced inventory in the inter-dealer market, that will be open for negotiation in two consecutive rounds of trade. We identify the first inter-dealer stage as bilateral direct trades between dealers.

Winning dealers will have a final opportunity to trade in a subsequent round of trade, an indirectly inter-dealer broker system that we believe captures in a certain way the inter-dealer broker's screens. Anonymous inter-dealer broker systems are specially popular among government securities, accounting for more than 90% of the inter-dealer US Treasuries market<sup>7</sup>. Both rounds will be informative of the asset's final payoff for the remaining dealers in the marketplace, although this information revealing process will be modeled here as noisy signals, not standard in the microstructure literature, as we believe this captures in a parsimonious way the idea that players revise their beliefs based on others' actions and that it may take more than one period to invert the full information content. The precision of the noisy signal will be made both exogenous and endogenous. We run simulations showing that when we let the volume traded in the first inter-dealer stage defines the precision of the informative signal in the final stage of trade, dealers will play strategically to avoid revealing information. This will restrict the ability of the market makers to trade in the initial public round of trade and decrease the risk-sharing in first place.

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